Representation Theory — WiSe 2022

Exercises before the exam

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1. Show that any irreducible complex representation of a finite abelian group is one dimensional. Give an example where this is false if the field is not algebraically closed. 5 P.

2. Suppose that G is a finite group and C is a conjugacy class in G. Show that $g_C = \sum_{g \in C} g$ acts on any irreducible representation of G by a scalar. What is this scalar? 5 P.

3. Consider the irreducible representations of S_4. 15 P.

(i) Which are still irreducible when restricted to A_4?
(ii) Is this map injective on isomorphism classes of irreducible representations?
(iii) Which irreducible representations of A_4 arise from this process?

4. State and prove *Schur's Lemma* for irreducible representations of algebras over an arbitrary field, and its specialization to algebraically closed fields. 5 P.

5. Let A be a finite dimensional algebra and let M be a right A-module. Show that there is a (natural) isomorphism $\operatorname{soc}(DM) \longrightarrow D(M/\operatorname{rad} M)$. 3 P.

6. Given an example of a finite dimensional algebra A and a non-simple finite dimensional A-module M for which $\operatorname{End}_A(M)$ is a division algebra. 2 P.

7. Give the two definitions of the radical of a module category, show that they are ideals, and show that they are equivalent. 20 P.

8. Consider the A_n quiver, with vertices $1, 2, \ldots, n$ and an arrow $i \to i+1$ for each $i \in \{1, \ldots, n-1\}$. Compute τ on every simple right module module. 10 P.

9. Let P be indecomposable and projective. Show that $\operatorname{rad} P \longrightarrow P$ is right minimal almost split. Conclude that if $M \longrightarrow P$ is irreducible then M is a summand of $\operatorname{rad} P$. 8 P.